Spin-$\frac{1}{2}$ transverse XX chain with a correlated disorder: dynamics of the transverse correlations

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Abstract

We examine numerically the dynamics of zz correlations in the spin-$\frac{1}{2}$ isotropic XY chain with random intersite coupling and on–site transverse field that depends linearly on the neighbouring couplings (correlated off–diagonal and diagonal disorder). We discuss the changes in the frequency profiles of zz dynamic structure factor caused by disorder.

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Recently the properties of the spin-$\frac{1}{2}$ transverse XX chain with correlated disorder have been discussed in some detail [1, 2]. Such a model consists of $N \to \infty$ spins $\frac{1}{2}$ on a circle governed by the Hamiltonian

$$H = \sum_{n=1}^{N} \Omega_n s_n^z + \sum_{n=1}^{N} J_n \left(s_n^x s_{n+1}^x + s_n^y s_{n+1}^y \right). \tag{1}$$

It is assumed that the intersite couplings J_n are independent random variables each with the probability distribution $p(J_n)$ and the on–site transverse field Ω_n is determined by the surrounding couplings J_{n-1} and J_n according to the formula

$$\Omega_n = \overline{\Omega} + \frac{a}{2} \left(J_{n-1} + J_n - 2\overline{J} \right) \tag{2}$$

where $\overline{\Omega}$ and $\overline{J}$ are the mean values of Ω_n and J_n, respectively, and a is a real parameter. The models with correlated disorder should arise while describing materials with topological disorder. Although there are a few examples of real materials which are reasonably well described by the one–dimensional spin-$\frac{1}{2}$ isotropic XY model (see, for example, [3]) the introduced model (1), (2) to our best knowledge was not related to any particularly compound. However, it is still of much use for understanding the generic effects of disorder since in the case of the Lorentzian probability distribution $p(J_n)$ and $|a| \geq 1$ it is possible to find explicitly the exact expression for the random–averaged density of states $\overline{\rho(E)}$ ($\overline{(\ldots)} \equiv \ldots \int_{-\infty}^{\infty} dJ_n p(J_n) \ldots (\ldots)$) and thus to examine rigorously the thermodynamic properties of a magnetic model with randomness [1].

The obtained up till now exact analytical results pertain only to thermodynamics. In the present paper we study the effects of correlated disorder on the dynamics of spin correlations examining for this purpose the zz dynamic structure factor

$$\overline{S_{zz}(\kappa, \omega)} = \int_{-\infty}^{\infty} dt \mathrm{e}^{-\epsilon|t|} \mathrm{e}^{i\omega t} \sum_{n=1}^{N} \mathrm{e}^{i\kappa n} \left[\overline{\langle s_j^z(t) s_{j+n}^z \rangle} - \overline{\langle s_j^z \rangle \langle s_{j+n}^z \rangle} \right], \quad \epsilon \to +0. \tag{3}$$

The evaluation of the zz time–dependent spin correlation functions $\overline{\langle s_j^z(t) s_{j+n}^z \rangle}$ cannot be performed analytically but it can be done numerically [4] (see also [5, 6, 7, 8]). In what follows we

consider the rectangle (but not Lorentzian) probability distribution

$$p(J_n) = \frac{1}{2\Delta}\Theta(J_n - \overline{J} + \Delta)\left[1 - \Theta(J_n - \overline{J} - \Delta)\right], \tag{4}$$

where Δ controls the strength of disorder. From (2), (4) one can find the probability distribution for the random variable Ω_n

$$p(\Omega_n) = \frac{1}{|a|\Delta}\left(1 - \frac{1}{|a|\Delta}|\Omega_n - \overline{\Omega}|\right)$$
$$\times\Theta(\Omega_n - \overline{\Omega} + |a|\Delta)\left[1 - \Theta(\Omega_n - \overline{\Omega} - |a|\Delta)\right]. \tag{5}$$

To reveal the effects of correlated disorder besides the model defined by (1), (4), (2) we consider the case of non–correlated disorder for model (1) assuming that J_n and Ω_n are independent random variables with probability distributions (4) and (5), respectively. In our computations we considered chains of $N = 400$ spins with $\overline{J} = -1$, $\overline{\Omega} = 0.5$ at low temperature $\beta = 1000$. We took $\Delta = 0.5$ for correlated disorder with $a = \pm 1.01$ and for non–correlated disorder and performed the random averaging of the zz dynamic correlation functions over 3000 random realizations. We put in (3) $j = 150$ and computed correlation functions with n up to 100 for the times up to $15, \ldots, 160$ (depending on the value of κ). We adopted $\epsilon = 0.001$. To prove that our results for the taken values of parameters already pertain to thermodynamic systems we performed many additional calculations similar to that described in [4]. The main results of our study are shown in Fig. 1 where we displayed the frequency dependence of $\overline{S_{zz}(\kappa,\omega)}$ at $\kappa = \frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$ for different types of disorder.

Let us turn to a discussion of the obtained results. Dynamics of the transverse correlations in the non–random case is well known [9, 10, 11]. In the Jordan–Wigner picture the zero–temperature zz dynamic properties of the spin-$\frac{1}{2}$ transverse XX chain are conditioned by exciting of two fermions with energies $\Lambda_{\kappa'} = \Omega + J\cos\kappa' < 0$ and $\Lambda_{\kappa''} = \Omega + J\cos\kappa'' > 0$, for which $\omega = -\Lambda_{\kappa'} + \Lambda_{\kappa''}$ and $\kappa'' = \kappa' - \kappa$. Consider, for example, $S_{zz}(\frac{\pi}{4},\omega)$ (dashed curve in Fig. 1a). Evidently, $-\frac{\pi}{3} < \kappa' < -\frac{\pi}{12}$, $-\Lambda_{\kappa'} + \Lambda_{\kappa'-\frac{\pi}{4}} = -2\sin\frac{\pi}{8}\sin\left(\kappa' - \frac{\pi}{8}\right)$ and hence the lower frequency at which $S_{zz}(\frac{\pi}{4},\omega)$

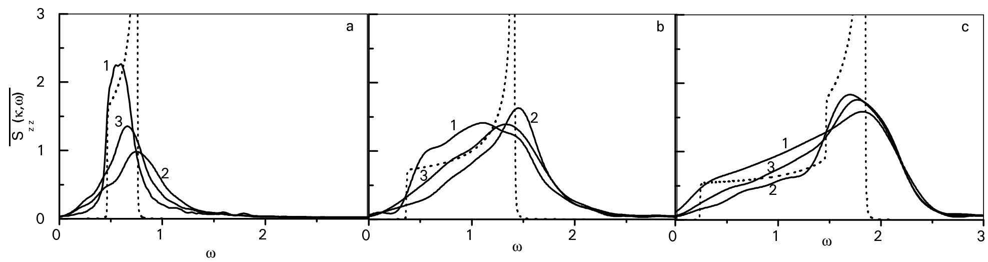

Figure 1: Frequency dependence of $\overline{S_{zz}(\kappa,\omega)}$ at $\kappa=\frac{\pi}{4}$ (a), $\kappa=\frac{\pi}{2}$ (b), $\kappa=\frac{3\pi}{4}$ (c) at low temperature $\beta=1000$ for model (1) with $\overline{J}=-1$, $\overline{\Omega}=0.5$, $\Delta=0.5$; 1 — correlated disorder with $a=-1.01$, 2 — correlated disorder with $a=1.01$, 3 — non–correlated disorder. Dashed curves correspond to the non–random case.

appears is equal to ≈ 0.466 (two fermions with the energies $\Lambda_{-\frac{\pi}{12}}\approx-0.466$ and $\Lambda_{-\frac{\pi}{3}}=0$, respectively), the upper frequency after which $S_{zz}(\frac{\pi}{4},\omega)$ disappears is equal to ≈ 0.759 (two fermions with the energies $\Lambda_{-\frac{\pi}{3}}=0$ and $\Lambda_{-\frac{7\pi}{12}}\approx 0.759$, respectively). We may relate the changes in the transverse dynamic structure factor due to randomness to the changes in the random–averaged density of states for different types of disorder (Fig. 2). Indeed, the pair of fermions determining the lower frequency roughly speaking does exist for $a=-1.01$ and does not exist for $a=1.01$ and for non–correlated disorder, whereas the density of states for the energies corresponding to the pair of fermions determining the upper frequency is diminished equally because of disorder in all three cases.

Consider further $S_{zz}(\frac{\pi}{2},\omega)$. Repeating the above arguments one concludes that two fermions with the energies $\Lambda_{\frac{\pi}{6}}\approx-0.366$ and $\Lambda_{-\frac{\pi}{3}}=0$ determine the lower frequency ≈ 0.366, starting from the frequency ≈ 1.366 two pairs of fermions contribute to the transverse dynamic properties (e.g., at that frequency one finds two fermions with the energies $\Lambda_{-\frac{\pi}{6}}\approx-0.366$ and $\Lambda_{-\frac{2\pi}{3}}=1$ and another pair of fermions with the energies $\Lambda_{-\frac{\pi}{3}}=0$ and $\Lambda_{-\frac{5\pi}{6}}\approx 1.366$), and two fermions with the energies $\Lambda_{-\frac{\pi}{4}}\approx-0.207$ and $\Lambda_{-\frac{3\pi}{4}}\approx 1.207$ determine the upper frequency ≈ 1.414. Analysing $\overline{\rho(E)}$ in Fig. 2 one observes, for example, that $\overline{\rho(E)}$ at the energies related to the lower frequency is almost not diminished for the correlated disorder with $a=-1.01$, whereas $\overline{\rho(E)}$ is diminished

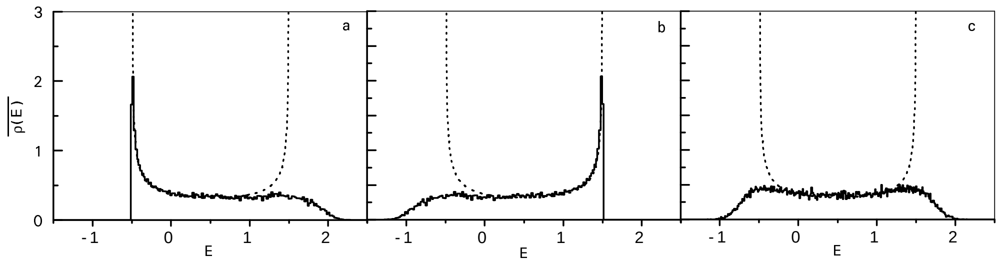

Figure 2: $\overline{\rho(E)}$ evaluated numerically (see [4]) for model (1) with $\overline{J}=-1$, $\overline{\Omega}=0.5$, $\Delta=0.5$; a — correlated disorder with $a=-1.01$, b — correlated disorder with $a=1.01$, c — non–correlated disorder. Dashed curves correspond to the non–random case.

essentially for $a=1.01$ and non–correlated disorder. This is in agreement with the changes in $\overline{S_{zz}(\frac{\pi}{2},\omega)}$ seen in Fig. 1b.

Consider finally $S_{zz}(\frac{3\pi}{4},\omega)$. Similarly to the previous cases one finds that the lower frequency ≈ 0.241 is conditioned by two fermions with the energies $\Lambda_{\frac{\pi}{3}}=0$ and $\Lambda_{-\frac{5\pi}{12}}\approx 0.241$, starting from the frequency ≈ 1.466 $S_{zz}(\frac{3\pi}{4},\omega)$ is determined by two pairs of fermions (e.g., at that frequency one finds two fermions with the energies $\Lambda_{\frac{\pi}{12}}\approx -0.466$ and $\Lambda_{-\frac{2\pi}{3}}=1$ and another pair of fermions with the energies $\Lambda_{-\frac{\pi}{3}}=0$ and $\Lambda_{-\frac{13\pi}{12}}\approx 1.466$), and the upper frequency ≈ 1.848 is conditioned by two fermions with the energies $\Lambda_{-\frac{\pi}{8}}\approx -0.424$ and $\Lambda_{-\frac{7\pi}{8}}\approx 1.424$. From Fig. 2 one notes that in the case of lower frequency the disorder decreases $\overline{\rho(E)}$ at the corresponding energies for non–correlated disorder and the correlated one with $a=1.01$ stronger than for the correlated disorder with $a=-1.01$, whereas in the case of the upper frequency $\overline{\rho(E)}$ at the corresponding energies is diminished more for non–correlated disorder than for correlated disorder that is consistent with frequency profiles seen in Fig. 1c.

To summarize, we examined the low–temperature dynamics of the transverse spin correlations in the spin-$\frac{1}{2}$ transverse XX chain with correlated disorder computing the transverse dynamic structure factor $\overline{S_{zz}(\kappa,\omega)}$. We found that within certain frequency regions the introducing of disorder may yield almost no changes in the value of $\overline{S_{zz}(\kappa,\omega)}$ (e.g., $\overline{S_{zz}(\frac{\pi}{4},\omega)}$ at $\omega\approx 0.5$ for $a=-1.01$ or $\overline{S_{zz}(\frac{\pi}{2},\omega)}$ at $\omega\approx 1$ for non–correlated disorder). We observed that the changes in

$\overline{S_{zz}(\kappa,\omega)}$ caused by disorder may be explained by the changes in the random–averaged density of states. Evidently, because of the Jordan–Wigner mapping the obtained results may be useful for understanding the conductivity in a chain of tight–binding fermions with random correlated hopping and on–site energy.

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